Spectroscopy in Motion: A Method to Measure Velocity

Did you ever hear a train whistle or truck on a highway as it approaches you and then passes at a high rate of speed? Go to and listen to the Doppler shift for sound waves at http://www.colorado.edu/physics/2000/applets/doppler2.html. Describe the sound.

Spectra and the Doppler Shift

Now we are going to use the Doppler shift with light waves to see how it influences the lines of atomic spectra. You will need to go to the interactive Excel spreadsheet at http://academic.pgcc.edu/~ssinex/excelets/spectra_2.xls. Open the spreadsheet and go to the Doppler tab, which shows the atomic spectrum for hydrogen. How do the spectral lines behave for a velocity moving toward the observer and away from the observer? You may want to turn the stationary reference lines “on” using the check box. Illustrate on the diagram below.

Can you tell an approaching object from one moving away from you? Explain.
The difference in position of the lines for the moving spectra is due to the Doppler shift. The spectral lines of objects moving away from an observer are shifted toward the red end of the visible spectrum and are said to be red-shifted; while objects moving toward an observer are said to be blue-shifted. An object moving parallel with the observer would NOT show a Doppler shift in the spectrum, motion must be away from or toward the observer along the line of sight (line connecting observer with object).

Using spectroscopy, the velocity \( v \) of the moving object can be determined from its red shift (or blue shift). The equation holds only if \( v \ll c \) or \( z < 1 \) (see below).

\[
v = \left( \frac{\Delta \lambda}{\lambda} \right) c
\]

where \( \Delta \lambda \) is the shift in wavelength, \( \lambda \), and \( c \) is the velocity of light. Red shifted objects will have a positive velocity, while blue shifted velocities will be negative. This velocity is called the radial velocity.

Astronomers use the symbol \( z \) to represent the red shift or

\[
z = \frac{\Delta \lambda}{\lambda}
\]

These calculations are set-up on the Doppler tab if you select the "show data and results" check box.

How much of a shift, \( \Delta \lambda \), occurs for the each of the hydrogen spectral lines at 486 nm if \( z = 0.13 \) as measured for galaxies in Bootes? How about the line at 656 nm?
Redshift values of $z = 4$ have been measured for some distant quasars found at the edge of the universe. However, the velocity must be determined using the relativistic form of the equation when $z > 1$. The relativistic form is used as velocities approach the speed of light, since behavior is described by the theory of relativity.

Nothing can go faster than the speed of light ($c$), 300,000 km/sec. This is the cosmic speed limit! Consider the plot of redshift, $z$, against $v/c$ ratio, where the ratio is corrected for relativistic effects.

The earlier equation, $z = v/c$ is shown on the graph above. Where does the data agree well with the equation? Where does it diverge?

The data points fall on this line when $z < 1$. However, as $z > 1$ the data points are considerably away from the $z = v/c$ line. The data points curve to approach the cosmic speed limit, the speed of light.

Using special relativity developed by Einstein, the relativistic version of the equation relating redshift to velocity is given by:

$$z = \frac{1 + v/c}{1 - v/c} - 1$$

This equation above simplifies to the nonrelativistic form, $z = v/c$, when $v \ll c$. 
On the *relativity tab*, you get to explore error of the nonrelativistic equation compared to the relativistic version. At what value of $z$ does the error reach 10%? Use the slider (scroll bar) above the graph to determine value.

The largest redshift measured for the galaxy RD1 is 5.34 (Science, 13 March 1998). Determine the $v/c$ ratio for RD1 by either using the relativistic equation below or use the graph on the *relativity tab* to estimate it.

$$\frac{v}{c} = \left( \frac{(z + 1)^2 - 1}{(z + 1)^2 + 1} \right)$$

The RD1 galaxy has a velocity of 0.95$c$. If the 122 nm hydrogen line, which is considered in the far ultraviolet, was detected in RD1, what would its $\Delta \lambda$ be?

What part of the electromagnetic spectrum would the line appear in after the shift? Some common wavelength regions and their wavelengths are given in the table below. Remember that $z = \Delta \lambda / \lambda$.

<table>
<thead>
<tr>
<th>Region</th>
<th>Wavelength (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ultraviolet</td>
<td>10 - 400</td>
</tr>
<tr>
<td>Visible</td>
<td>400 - 700</td>
</tr>
<tr>
<td>Infrared</td>
<td>700 - 100,000</td>
</tr>
</tbody>
</table>
Edwin Hubble essentially discovered that most objects in the universe are moving away from each other. This indicates the expansion of the universe. Determining Hubble’s Law is a major use of redshift data for a variety of astronomical objects.

Hubble’s Law shows that the radial velocity is a linear function of the distance the object is from Earth, or, the farther an object is from the Earth, the faster it recedes away. The radial velocity is determined by the redshift of spectral lines.

\[ v = H_0 \cdot d \]

where \( v \) is the radical velocity in km/sec, \( d \) is the distance in Mpc, and \( H_0 \) is the Hubble constant in km/sec-Mpc. Astronomers then can use this as a distance measuring tool. To get accurate distances, we need an accurate value for the Hubble constant. Now go to the Hubble tab to address the following questions.

How would you describe the pattern of Hubble’s original 1929 data on the plot?

How would you describe Hubble’s later data?

What were the two early values of the Hubble constant?

Later data show a somewhat different picture. Describe the data and graph. What has changed about the graph and the Hubble constant?

From the velocity determined earlier for RD1 and newer value of the Hubble constant, how far is RD1?
The reciprocal of the Hubble constant, $H_0$, with some unit conversions will yield an estimate of the age of the universe (see *Hubble tab*). How has the age estimate changed from Hubble’s early value to now?

Another practical application of the Doppler shift is for measuring the rotation velocity of stars. Extremely young B stars spin at their equator at 300 km/sec, while our Sun rotates at 2 km/sec.

How do you think the spectral lines will appear in the rotating star’s spectrum?

See the *rotation tab* to increase stellar spin. Illustrate it on the diagram below.
If you were observing the star directly above its spin axis, would you see a Doppler shift? Explain.

The total width of a spectral line increases with increasing rotational velocity of a star.

Check your understanding
The middle spectrum below shows an object moving away at 10,000 km/sec compared to the stationary spectrum (top). On the lower diagram carefully illustrate the spectrum for the object moving away at 20,000 km/sec.
What next?
The discovery of extrasolar planets or exoplanets has been aided due to recent advances in spectrometers with very high resolutions able to measure extremely small wavelength shifts. From the variation in a star's Doppler shift caused by an orbiting planet, the period of the planet, its orbital radius and velocity, and minimum mass can be determined. Most spectrometers in use for searching for extrasolar planets claim a detection down to 10 m/s. In nanometers, this is an incredibly small shift, but it can be done! More about exoplanets is given in Spectroscopy of Motion II.