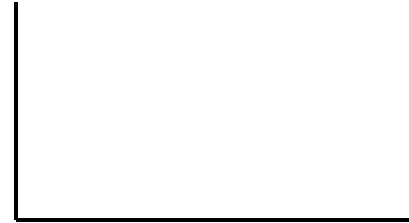


## Developing a Mathematical Model for a Burning Candle

If you were to collect the height data of a burning candle over time, how would the data plot? Sketch and label a graph.



### The Virtual Burn

We will accomplish our burning the candle experiment in the safety of the virtual realm via an interactive Excel spreadsheet or Excelet. Bring up the Excelet at [http://academic.pgcc.edu/~ssinex/excelets/burning\\_candles.xls](http://academic.pgcc.edu/~ssinex/excelets/burning_candles.xls). The screen shot shows the "burn & observe" tab. Here we can set the height of two different candles and then collect data by clicking the up arrow of the spinner to increase time passing to observe what will happen. We can choose from four different experiments to run.

**Burning a Candle** Click to have time pass and collect data

time passing =  ▲  
to restart type in zero ▼

starting candle 1  
height =  cm

starting candle 2  
height =  cm

height =  cm      height =  cm

As time passes, what happens to the candle?

Height of candle

| Candle   | Height (cm) |
|----------|-------------|
| Candle 1 | 10          |
| Candle 2 | 5           |

Height, cm

time

| time | candle 1 (cm) | candle 2 (cm) |
|------|---------------|---------------|
| 0    | 10            | 5             |

Experiment 1     Experiment 2  
 Experiment 3     Experiment 4

As time passes, how would you describe the data for a candle burning? Sketch a set of data on the graph above.

How does your prediction above compare to the result?

How do you determine the burn rate for the candle? What are the units of the burn rate?

What does the y-intercept on the graph represent?

Now go to the "adjust burn" tab.

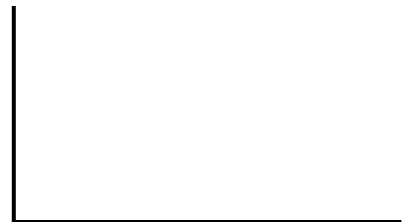
If you adjust the burn rate, how does the graph respond?

As time passes for a set of data, how would you describe the behavior of the burn rate? If you had to graph burn rate as a function of time what would it look like?

What does the value of the x-intercept represent?

Write an equation in terms of the variables studied (not x and y) to describe the burning candle model.

If you had two candles of different height with the same burn rates, how would the data compare? Sketch a set of axes with both candles shown on it.



### Experimental Error

Let's consider some real-world data where there would be some experimental error involved with the data collection. Go to the "error" tab. Here we are showing a line of best-fit determined by linear regression. We can judge the goodness-of-fit

of the regression line to the data by using the  $R^2$  value. A perfect fit, no error, has an  $R^2 = 1.00$ .

Now add some random error by dragging the scroll bar to the right or clicking the right arrow to increase the amount of error. As random error is added, what happens to the data on the graph? Describe it!

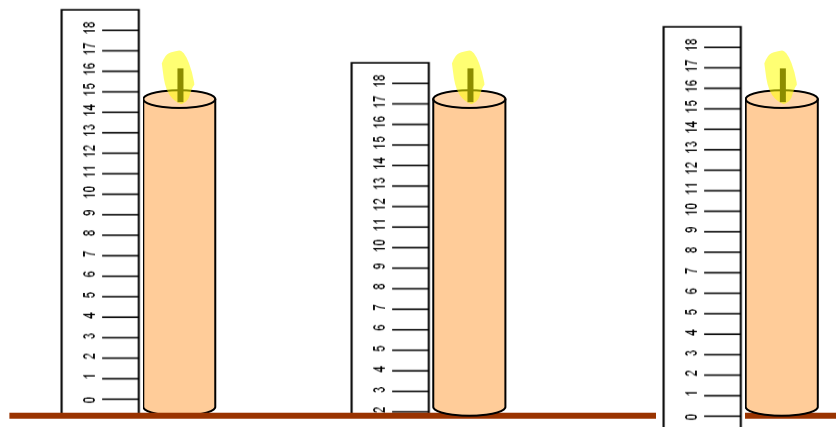
Vary the random error a small amount back and forth after setting some mid-range amount. How does it influence the best-fit line? Explain.

In general as random error is added, how does the  $R^2$  value respond?

Set the random error back to none and now investigate the systematic error. How does a positive systematic error influence the data?

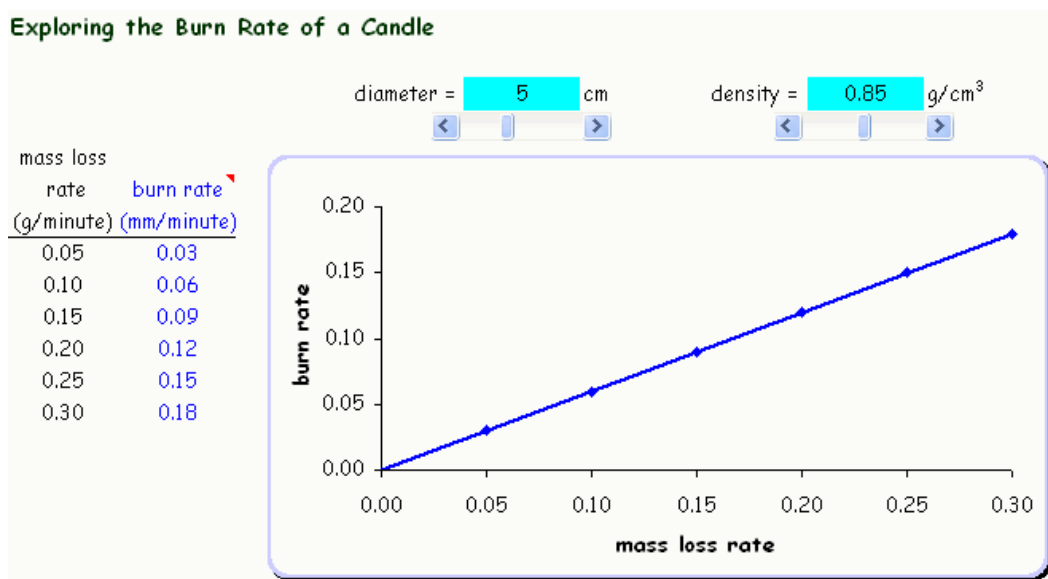
How does a negative systematic error influence the data?

Consider the candles and rulers below. What type of error occurs if any?



## Extending the Model to More Variables

The kinetics of the burning of a candle is a zero-order process since the height versus time plot is linear (slope constant). The rate of burning (-slope) is constant as the height changes over time. Let's explore the burning rate in more detail. This will extend our simple model to a multivariable model. We determined the burn rate in mm/time from the plot of height versus time. You could also measure the mass of the candle over time using a balance. This would give you the mass lost rate in g/time. The screen shot below shows how the burn rate is related to the mass loss rate. Now we can explore how the candle diameter and the density of the candle wax influence these parameters.



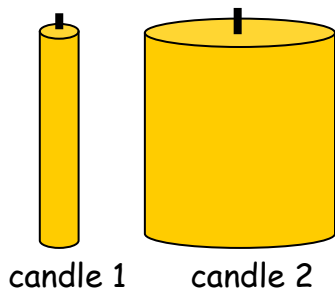
Click on the "explore burn rate" tab to start. You may want to turn on the comparison line to make this easier to visualize the effects.

How does the candle's diameter influence the burn rate?

How does the density of the candle wax influence the burn rate?

How does the temperature influence the density and hence, the burn rate?

Sketch and label a graph for the two candles, both with the same initial height, below.



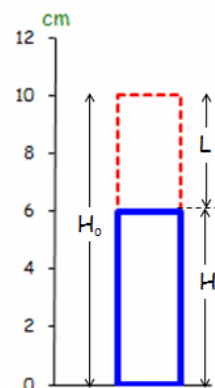
Which candle has the larger x-intercept? Why?

Let's explore one more variable by going to the "length burned" tab. Here we consider the length of the candle burned (the part that vaporized).

What type of relationship does the length burned as a function of time show?

Is the length burned as a function of time function the inverse function of the height as a function of time function? Explain why or why not.

Using the diagram to the right convert the height as a function of time function to the length burned as a function using algebra.



The "assessment" tab has some more questions to address.

### Notes to the Instructors

Here we develop a linear mathematical model with a positive y-intercept and a negative slope, since we are measuring the candle that remains over time. Our model is given mathematically as:

$$\text{Height (mm)} = -\text{slope (mm/time)} \times \text{time} + \text{initial height (mm)}$$

where the burn rate = - (-slope). The burn rate is then expressed as a positive value with units of mm/time. This is the rate of the disappearance of the candle (vaporized). The y-intercept is the initial height of the candle.

The x-intercept is the time when the height reaches zero (x-intercept = -initial height/-slope). It is the total time the candle will burn.

The introduction of random error causes scatter in the data points and the slope and y-intercept will vary as well. The  $R^2$  value will decrease as the random error increases (more scatter). Systematic error, mainly from the ruler (especially zero not an end of ruler) shifts the y-intercept only.

The burn rate is influenced by the following variables.

| Variable                     | Influence on burn rate  |
|------------------------------|---|
| Diameter of candle increases | Burn rate decreases   |
| Density of wax increases     | Burn rate decreases   |
| Temperature increases        | Volume expands, density decreases, hence burn rate increases* |

\*temperature can be important when you initially ignite the candle

So candle 2 will have a lower burn rate than candle 1 due to the large diameter.

The length burned is a direct relationship. The length burned versus time function is not the inverse function of the height versus time function. To be an inverse function, a relationship must be a reflection about the  $y = x$  function and this is not.

For a real experimental approach, see Sanger and others, Rate Law Determination of Everyday Processes, *J. Chem. Educ.* **79** (8), 989-991 (2002).