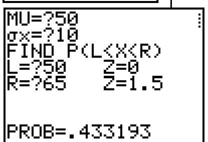
== **Begin 6.3** == 1 / 8 / 2008

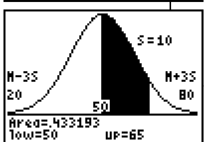
1 Corresponding Areas under a normal curve and the standard normal curve are equal. Below: Area B = Area A.

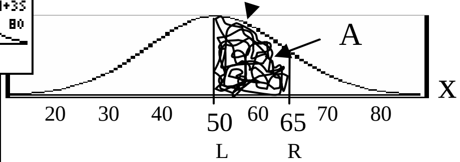
PRGM **NORMAL**
2
MU =50
σX=10
1
3:P(L<X<R)
L=50
R=65

A normal distribution		The standard normal distribution
$\mu = 50$	→	$\mu = 0$
$\sigma = 10$	→	$\sigma = 1$
$x_1 = 50$	→	$z_1 = 0$
$x_2 = \mathbf{65}$	→	$z_2 = \mathbf{1.5}$
$P(50 < x < 65) = .4332$	←	$P(0 < z < 1.5) = .4332$

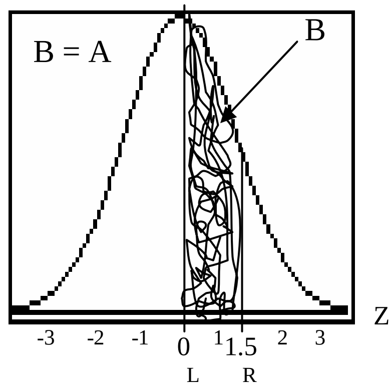
PRGM **NORMAL**
1
1
3:P(L<Z<R)
L=0
R=1.5







←



$z = \frac{x - \mu}{\sigma}$

$z = \frac{65 - 50}{10}$

$z = 1.5$

$z(65) \rightarrow 1.5$

$z(50) = 0$

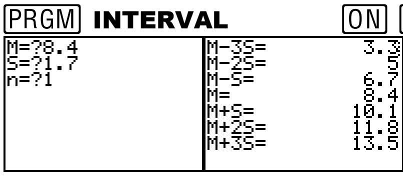
A normal distribution x has $\mu = 50$ and $\sigma = 10$. Find $P(50 < x < 65)$.

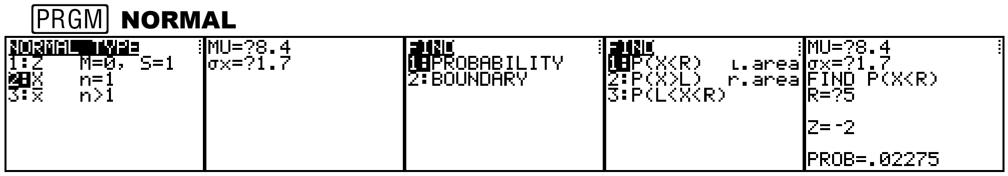
The traditional method is to convert the x values 50 and 65 to their corresponding z scores 0 and 1.5 using the z-score formula. Next, use the areas of the standard normal table to find A(0) = .5000, A(1.5) = .9332 .9332 – .5000 = .4332 → P(0 < z < 1.5) → P(50 < x < 65) = .4332

Page 332, 26b Given a ND with $\mu = 8.4$ and $\sigma = 1.7$. Let x = the police response time (in minutes).

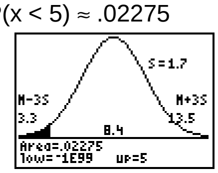
(b) Find the probability that the police respond within 5 minutes. That is find $P(x < 5)$.

PRGM **INTERVAL** ON 1 PRGM **NORMAL**

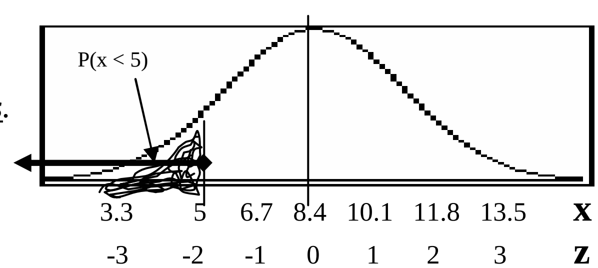




$P(x < 5) \approx .02275$



Draw the graph, showing intervals.



By table: z(5) = -2.
A(-2) = .0228
P(z < -2) ≈ .0228
P(x < 5) ≈ .0228

About 2% of the time, the response time is less than 5 minutes.

==

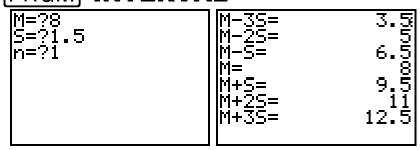
Page 333, 31d Given $\mu = 8$. When σ is not given, use <u>the general rule</u>: $\boxed{\sigma \approx \frac{\text{Range}}{4}} = \frac{11-5}{4} = 1.5$.

Let x be the replacement age of a TV set.

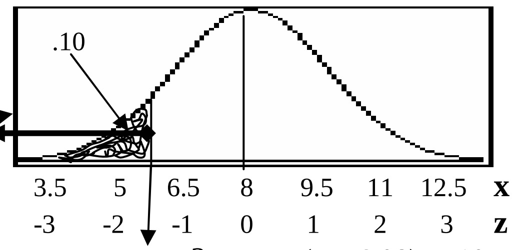
(d) Find x_0, <u>the guaranteed age</u> of a TV, so that no more than 10% of the sets are in the guarantee period.

That is, find x_0 so $P(x < x_0) \approx .10$.

PRGM **INTERVAL**



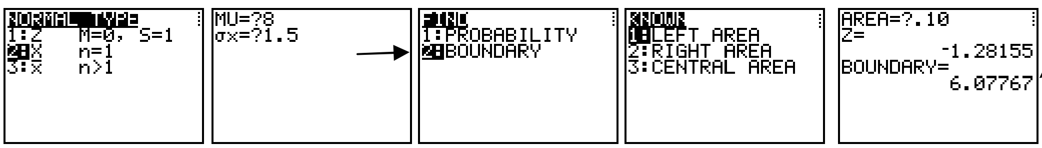
Draw the graph on paper.



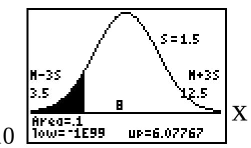
x_o = ? $P(x < 6.08) \approx .10$

Use the **NORMAL** *program to find <u>the boundary</u> of the known <u>Left Area</u>.*

PRGM **NORMAL**



$P(x < 6.08) \approx .10$



Conclusion:
If the company guarantees their sets for 6 years, they will not have to replace more than 10% of them.

<u>The Traditional Method</u>: ← <u>*Optional*</u> Find x_0 so $P(x < x_0) \approx .10$.

First find z_0 so $P(z < z_0) \approx .10$ using steps 1-4.

1 Draw left tail area C so that it contains 10% of the distribution. Mark the boundary z_0.

2 Look for .1000 in the <u>body</u> of the SNC table. .1000 is between .1003 and .0985.

3 Choose .1003 since it is closer to .1000.

4 The z-score that corresponds to .1003 is -1.28. → $z_0 = -1.28$.

Next find x Use $\boxed{x = z\sigma + \mu}$ to find x_0. → $x_0 = -1.28 \times 1.5 + 8 \approx 6.1$ So $P(x < 6) \approx .10$. $x_0 = 6$.

==

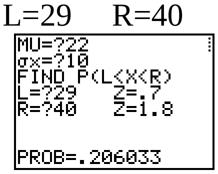
Page **332, 28** Given a ND with $\mu = 22$ and $\sigma = 10$. Let x = the daytime high temperature. Find $P(29 < x < 40)$.

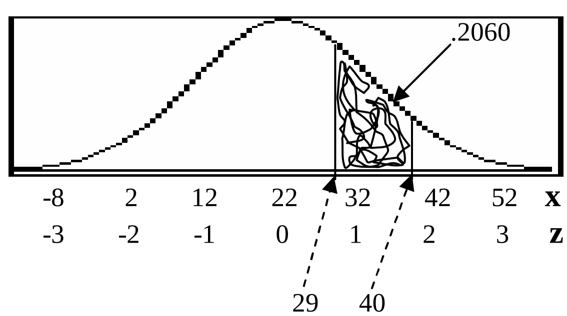
PRGM **NORMAL** 2 MU=22 σx=10 1 3 L=29 R=40

By table: z(29) = .7. z(40) = 1.8

A(.7) = .7580, A(1.8) = .9641

.9641 – .7580 = .2061





Result: PROB = .2060… $P(29 < x < 40) \approx .2060$

Conclusion:
The temperature is between 29°and 40° on about 21% of the days in January. $.206 \times 31 \approx 6$ days.
There are approximately 6 days in January when the daytime high temperature is between 29°and 40°.

==

== **Begin 7.1** ==

A Parameter <u>is</u> a number that is a descriptive measure of a <u>population</u>. *μ and σ are parameters.* *Page 360*

==

2 A Statistic <u>is</u> a number that is a descriptive measure of a <u>sample</u>. *$\bar{x}$ and s are statistics.* *Page 360*

==

3 A Sampling Distribution of $\bar{x}$ <u>is</u> **a probability distribution** formed by <u>all</u> the means $\bar{x}$ of <u>all</u> the samples, <u>all</u> of size n, from a population. This is called <u>the $\bar{x}$ distribution</u>. *See sheet 4.* *Page 361*

Consider all K samples of size n from a population x: S_1, S_2, S_3, …, S_K.
<u>The probability distribution</u> of all the means: $\bar{x}_1$, $\bar{x}_2$, $\bar{x}_3$, …, $\bar{x}_K$ is <u>the $\bar{x}$ distribution</u>.

==**7.2** ==

4 The Mean of the $\bar{x}$ Distribution is denoted $\mu_{\bar{x}}$. *Page 366*

----

5 The Standard Deviation of the $\bar{x}$ Distribution is denoted $\sigma_{\bar{x}}$. *Page 366*

$\sigma_{\bar{x}}$ is also called <u>The Standard Error of the Mean</u> and denoted <u>SE Mean</u>.

==

6 Three Properties of the $\bar{x}$ Distribution are *See sheet 11.* *Page 366*

a) If the population is normal, then <u>the $\bar{x}$ distribution</u> is normal. b) $\mu_{\bar{x}} = \mu$. c) $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$

Consider a population x with $\mu = 100$ and $\sigma = 20$. Let $\bar{x}$ denote <u>the $\bar{x}$ distribution</u> for samples of size n = 25.

a) If x is normal, then <u>the $\bar{x}$ distribution</u> is also normal. b) $\mu_{\bar{x}} = 100$. c) $\sigma_{\bar{x}} = \dfrac{20}{\sqrt{25}} = \dfrac{20}{5} = 4$.

==

(7) The Central Limit Theorem states: a) <u>The $\bar{x}$ distribution</u> from <u>any</u> population is *usually* <u>normal</u> when n > 29,

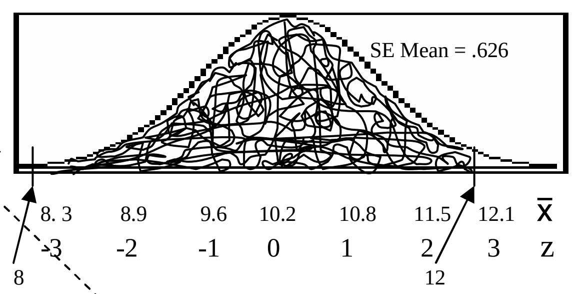
b) $\mu_{\bar{x}} = \mu$., and c) $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$ *See sheet 11.* *Page 368*

==

Page 366, 2b A normal population has $\mu = 10.2$ and $\sigma = 1.4$. Five fish are caught. → n = 5. ←

Let $\bar{x}$ be the mean (average) length of the 5 fish. Find P(8 < $\bar{x}$ < 12).

The $\bar{x}$ Distribution with n = 5



The $\bar{x}$ distribution has $\mu_{\bar{x}} = 10.2$ and $\sigma_{\bar{x}} = \dfrac{1.4}{\sqrt{5}} \approx .626$ = SE Mean.

PRGM **INTERVAL** M=10.2 S=1.4 **n = 5** → *Draw the graph* →

PRGM **NORMAL** 3 MU=10.2 σx=1.4 **n=5** 1 3 L= 8 R=12 .

Result: PROB ≈ .9978 P(8 < $\bar{x}$ < 12) ≈ 99.8%.

<u>The probability</u> is 99.8% that <u>the average length</u> of the 5 fish will be between 8 and 12 inches.

Note: <u>z(8)</u> ≈ -3.51. z(12) ≈ 2.87.

A(-3.51) = 0

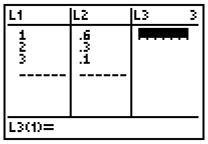
$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{8 - 10.2}{\frac{1.4}{\sqrt{5}}} = -3.51$$

```
MU=?10.2
σx=?1.4
n=?5
SE MEAN=.6261
P(L<x̄<R)
L=?8      Z=-3.514
R=?12     Z=2.875
PROB=.997759
```

PRGM **ZTXVALUE** M=10.2 S=1.4 **n=5** *Select* **4** . → SE Mean = .626 = $\sigma_{\bar{x}}$.

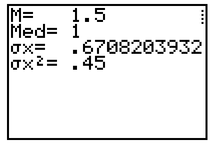
==

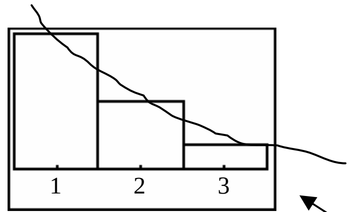
Consider the <u>population</u> x: 1,1,1,1,1,1,2,2,2,3.   N = 10, μ = 1.5, $\sigma \approx .671$



x	1	2	3
P(x)	.6	.3	.1

The Probability Distribution of x





The x Distribution

From the population, all samples of size n = 3 are selected *with replacement*. With replacement means that after each number is selected, it is returned to population *before* the next number is selected.

There are1000 samples. 27 are <u>different</u> samples.   There are 7 different values of $\bar{x}$

10×10×10   3×3×3

See sheet 10

The different samples

Sample S	$\bar{x}$	P(S)
1,1,1	1	.216
1,1,2	4/3	.108
1,1,3	***5/3***	***.036***
1,2,1	4/3	.108
1,2,2	***5/3***	***.054***
1,2,3	2	.018
1,3,1	5/3	.036
1,3,2	2	.018
1,3,3	7/3	.006
2,1,1	4/3	.108
2,1,2	5/3	.054
2,1,3	2	.018
2,2,1	5/3	.054
2,2,2	2	.027
2,2,3	7/3	.009
2,3,1	2	.018
2,3,2	7/3	.009
2,3,3	8/3	.003
3,1,1	5/3	.036
3,1,2	2	.018
3,1,3	7/3	.006
3,2,1	2	.018
3,2,2	7/3	.009
3,2,3	8/3	.003
3,3,1	7/3	.006
3,3,2	8/3	.003
3,3,3	3	.001
sum	→	1

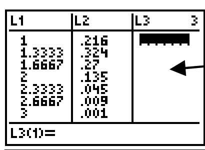
P(1,1,1) = .6×.6×.6 = .216
P(1,1,2) = .6×.6×.3 = .108
P(1,1,3) = .6×.6×.1 = .036

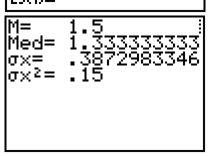
P(1,2,2) = .6 ×.3×.3 = .054

The $\bar{x}$ distribution

$\bar{x}$	P($\bar{x}$)	f
1	.216	1
4/3	.324	3
5/3	.270	6
2	.135	7
7/3	.045	6
8/3	.009	3
3	.001	1
sum	1	27

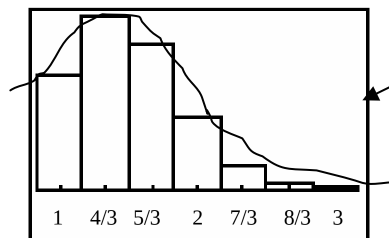
P(1) = .216
P(4/3) = .108+.108+.108 = .324
P(5/3) = .036+.054+.036+.054+.054+.054=.270
Note that P(5/3) ≠ 6/27 because values of 5/3 are not equally likely.





USE PRGM **PROBDIST**

$\mu_{\bar{x}} = 1.5$, $\sigma_{\bar{x}} \approx .387$



The $\bar{x}$ distribution

Checking the theory: $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{.671}{\sqrt{3}} \approx .387$ also, $\mu_{\bar{x}} = \mu$

PRGM **ZTXVALUE** M=1.5 S=.671 **n=3** *Select* 4 . ➔ SE Mean ≈ .387 = $\sigma_{\bar{x}}$.

===

==

Page 374, 8 Let x be (the white blood cell count) per (mm^3 of whole blood).

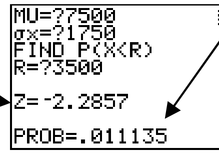
x is a normal distribution with $\mu = 7500$ and $\sigma = 1750$. (a) Find $P(x < 3500)$ for **n = 1**, a single blood sample.

Use PRGM **INTERVAL** *to help make the graph.*

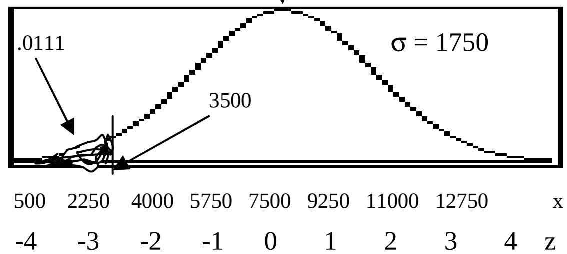
PRGM **NORMAL** 2 MU=7500 σx=1750 *1 1* **R** = 3500 — *Result:* PROB = .011135…

$P(x < 3500) \approx 1.1\%$

Note: z(3500) = –2.29.



$$z = \frac{x-\mu}{\sigma} = \frac{3500-7500}{1750} = -2.29$$



(b) Find $P(\bar{x} < 3500)$ for **n = 2**, for two blood samples.

PRGM **NORMAL** 3 MU= 7500 σx= 1750 **n= 2** *1 1* **R** = 3500

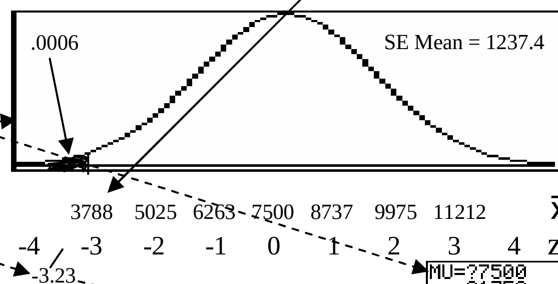
Use PRGM **INTERVAL** → M=7500 S=1750 n=2

Result: PROB = .000614

$P(\bar{x} < 3500) \approx .0006$

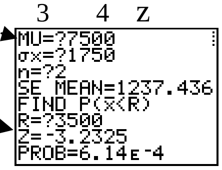
Draw this on paper.

Note: z(3500) = –3.23.



$$z = \frac{\bar{x}-\mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{(3500-7500)}{\left(\frac{1750}{\sqrt{2}}\right)} = -3.23$$

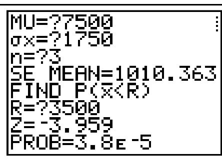
Use PRGM **ZTXVALUE** M=7500 S=1750 **n=2** 1

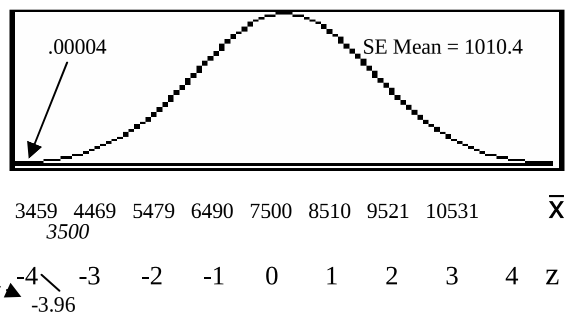


(c) Find $P(\bar{x} < 3500)$ for **n = 3**, for three blood samples.

PRGM NORMAL 3 MU=7500 σx =1750 **n=3** *1 1* **R** = 3500.

Use PRGM **INTERVAL** → M=7500 S=1750 n=3





$P(\bar{x} < 3500) \approx .00004 = .004\%$

Note: z(3500) = –3.96.

Use PRGM **ZTXVALUE** M=7500 S=1750 **n=3** 1 3500

== **8.1 & 8.2** ==

These sections develop the concept of a Confidence Interval for μ.

A confidence interval is an interval that is estimated to contain μ with a given level of confidence or reliability.

For example, you can say that μ is estimated to be in the interval (2, 4) with 99% confidence. $2 < \mu < 4$.

==

1 | The Confidence Level c | is the reliability of the confidence interval. *Page 401*

----

c is written as a percent. c = 95% or c = 99% are typical confidence levels.
c corresponds to a central area under a t curve found inside the back cover of our book.
A number cv called a critical value determines the central area c.

==

==

2 The Critical Value cv ,denoted z_c or t_c , is the right boundary of the central area c. *Page 401*

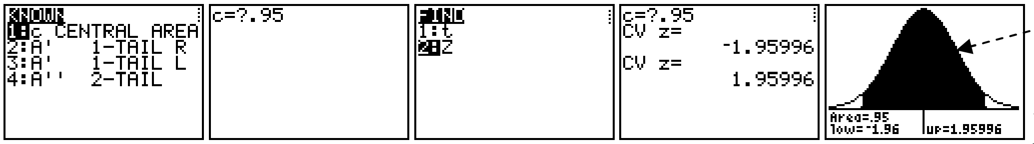
----

cv is found in the t table in the book (*inside back cover*). Use c in the top row and d.f. = n – 1 on the side.
z_c critical values are in the bottom row of the t table. t_c critical values are in the body of the table.

→ We will find cv using PRGM **cv**. ←

PRGM **cv**

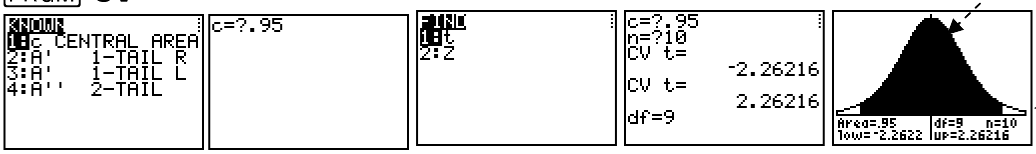
.950 = c



z

$z_{.95}$ = 1.96

PRGM **cv**



t

n = 10
$t_{.95}$ = 2.262

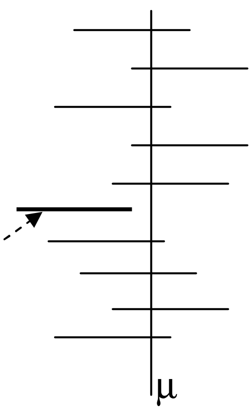
==

3 A c Confidence Interval for μ is an interval cCI that is estimated to contain μ with a c level of confidence.

Example: A 95%CI for μ is estimated to contain μ with a 95% level of confidence.

==

If we say that we are 90% confident that the 90% CI contains μ , we mean that if we have all of the 90% confidence intervals of all the samples of size n from the population, then μ would be in 90% of these intervals.



9 out of 10 of the 90% C intervals will contain μ.

One out of 10 of the 90% C intervals will not contain μ.

==

To find a confidence interval for μ we need: 1. a sample of size n, 2 $\bar{x}$ *called* A Point Estimate of μ
3. c, 4. σ or s, 5. cv, 6. the formula (or Interval function in the TI) for the cCI.

==

4 The cCI for μ when σ is given is $cCI = \left(\bar{x} \pm z_c \cdot \frac{\sigma}{\sqrt{n}}\right)$. This is the ZInterval. W*ith σ ,use* z_c.

==

5 The cCI for μ when σ is **not** given is $cCI = \left(\bar{x} \pm t_c \cdot \frac{s}{\sqrt{n}}\right)$. This is the TInterval. W*ith s, use* t_c.

Note: If the sample is small, then the population must be "normal" to use formula 4 or 5.

Here "normal" means: mound shaped, unimodal, and not extremely skewed.

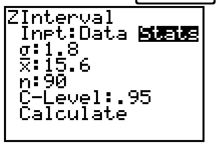
A Small Sample has n ≤ 29. A sample is small if n = 2, 3, … , 29.

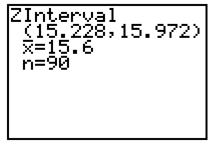
A Large Sample has n ≥ 30. A sample is large if n = 30, 31, 32, ….

==

Page 405, 2 $n = 90$, $\bar{x} = 15.60$, $\sigma = 1.80$. Find the 95% confidence interval for μ, her true average.
Since σ is given, a ZInterval is used.

STAT ◄ 7 *put cursor on* Calculate ENTER



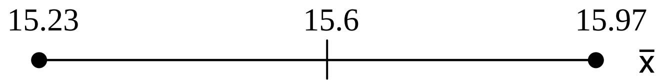


(Lower bound, Upper bound)

95% CI = (15.23, 15.97)

We are 95% confident that her average time μ is between 15.23 and 15.97 minutes.

A 95% CI for μ



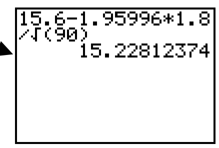
Optional Method

By formula: cCI = $\left(\bar{x} \pm z_c \cdot \frac{\sigma}{\sqrt{n}}\right)$. PRGM **cv** 1:c c = .95 2:Z . → 1.95996 = z_c.

$$95\%CI = \left(15.6 \pm 1.95996 \cdot \frac{1.8}{\sqrt{90}}\right) = (15.228, 15.97).$$

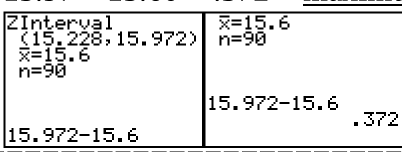
The cv of the 95% ZInterval.

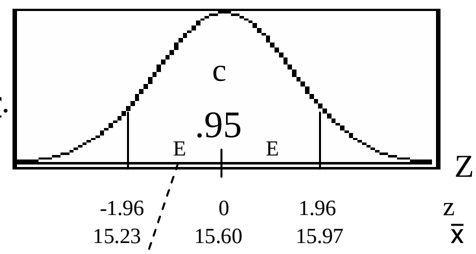
Keys for the lower bound: 15.6 − 1.95996 × 1.8 ÷ 2nd √ 90) ENTER → 15.228



Page 405, 2 continued

E = 15.97 – 15.60 = .372 = maximum error.



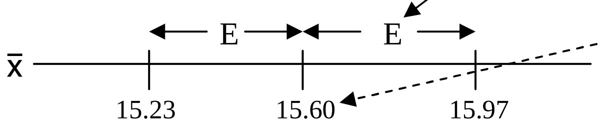


6 The Maximum Error of Estimate E of the cCI is $E = cv \cdot \frac{SD}{\sqrt{n}}$

$|\mu - \bar{x}|$ is called The Margin of Error.

In the problem above, $E = 1.95996 \cdot \frac{1.8}{\sqrt{90}} \approx .37$. The 95% CI would be (15.60 – .37, 15.60 + .37) = (15.23, 15.97)

15.60 ± .37



The center of the interval is always $\bar{x}$

Another way to find E, is to find the cCI, using the method at the top of the page, then subtract $\bar{x}$ from the upper bound. E = 15.97 – 15.60 = .37 min. .37×60sec = 22.2sec.

7 Student's t Distribution is given by $t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$ using $\bar{x}$ of all samples, of size n, from the population.

The Student's t distribution is used when σ is unknown. *The normal distribution z is used if σ is known.*

If n is small, $n < 30$, then the population must be "normal" to use the t distribution. *Page 413*

A t distribution has $\mu = 0$ and $\sigma = \sqrt{\frac{n-1}{n-3}}$. Every sample size n forms its own t distribution.

See **page 413** for examples of t graphs for n = 4 and n = 6. When $n = \infty$, (the t curve) = (the z curve).

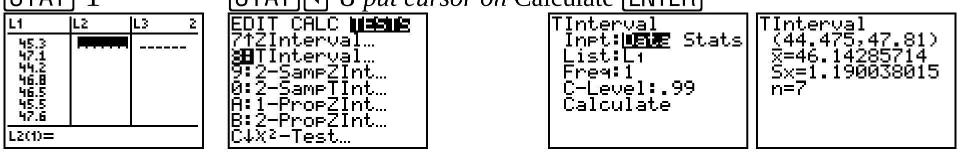
---

8 The Number of Degrees of Freedom d.f. for a t interval is the number d.f. = n – 1.

If n = 10, then d.f. = 10 – 1 = 9. *Also* n = d.f. + 1.

The equation (x + y + z = 20) has 2 degrees of freedom. We are *free* to select values for two of the variables, say x and y. Then, z must be computed. For example if we select x = 8 and y = 7, then z must be 5.
The equation (a + b + c + d + e = 10) has 4 degrees of freedom.

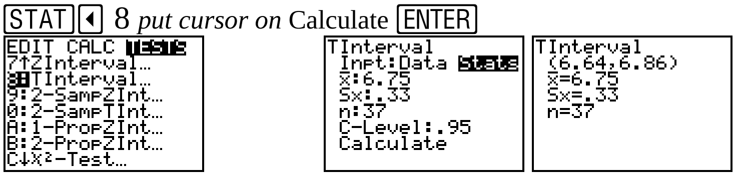
416, 4 STAT 1 STAT ◄ 8 *put cursor on* Calculate ENTER



cv = 3.707
E = 1.67

A 99% CI = (44.5, 47.8).

417, 3 STAT ◄ 8 *put cursor on* Calculate ENTER



cv = 2.028
E = .11

A 95% CI = (6.64, 6.86)

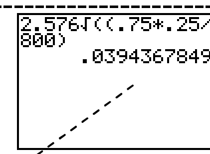
== Begin 8.3 ==

9 The Point Estimate $\hat{p}$ for p in a binomial experiment with n trials and r (x) successes is $\hat{p} = \frac{r}{n} = \frac{x}{n}$ *Page 426*

The point estimate for q is $\hat{q} = 1 - \hat{p}$. p is the probability of success or the proportion of successes.

A sample n = 800 students was taken from a population of 20000 students and given flu shots. Of the 800, 600 do not get the flu. We estimate the probability p that any student in the 20000 that got a shot will not get the flu to be $\hat{p} = \frac{600}{800} = 75\%$. $\hat{p}$ = P(will not get the flu| got a flu shot). The margin of error is $|p - \hat{p}|$.

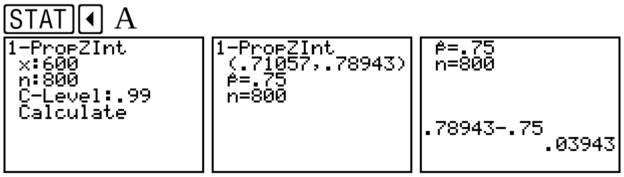
The Maximum Margin of Error E for a c level of confidence is $E = z_c \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$ *Page 426*

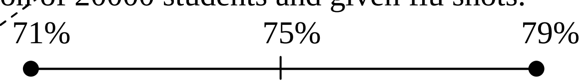


A c Confidence Interval for p is $\hat{p} \pm z_c \sqrt{\frac{\hat{p} \cdot \hat{q}}{n}}$ if $n\hat{p} > 5$ and $n\hat{q} > 5$.

428, 5d A sample of size n = 800 students is taken from a population of 20000 students and given flu shots. Of the 800, 600 do not get the flu. Find a 99% CI for p.

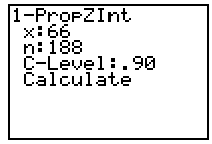
STAT ◄ A

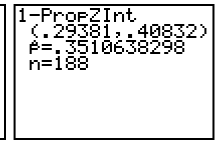




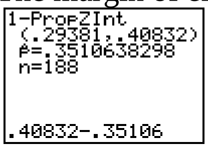
We are 99% confident that (71%, 79%) is one of the intervals that contains the actual value of p. That is, if all students got a shot, between 71% and 79% would not get the flu. We say this with 99% confidence. 75% ± 4% will not get the flu.

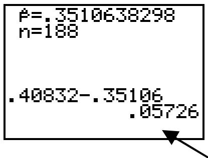
429, 4 n = 188, r = x = 66. Find a 90% CI for p. [STAT] [◄] A — A 90% CI = (29%, 41%)

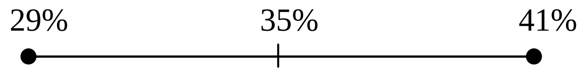




The margin of error is .40832 – .35106 is 5.7% ≈ 6%







35% of the books sold at a local bookstore are murder mysteries.
This estimate has a margin of error of 6%. ← *(often stated as ±6%)*

== **Begin 8.4** ==

10 The Sample Size n for a cCI for μ with <u>maximum error of estimate</u> E is $n = \left(\frac{z_c \cdot s}{E}\right)^2$ rounded up.

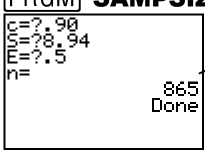
The formula for n above is derived from $E = z_c \cdot \frac{s}{\sqrt{n}}$ → $E\sqrt{n} = z_c \cdot s$ → $\sqrt{n} = \frac{z_c \cdot s}{E}$ ← square each side

==

Page 442, 2 Find <u>the sample size n</u> needed, to form a 90% CI for μ, for a maximum error of estimate E = .5.

After digging up at least 30 roots, s is found to be 8.94 in.

[PRGM] **SAMPSIZE**



Conclusion: We need the root lengths of 865 plants in order to be 90% confident that the mean of the 865 measurements will be within .5 inches of μ, the mean of the population of root lengths in YNP.

IF ,for example, $\bar{x}$ of the 865 plants is 10 inches, then our 90%CI with E = .5, would be (9.5, 10.5) inches.

The formula method : For a 90% CI the cv = z_c = 1.645 found on the bottom row of the t-table under c = .900.

$n = \left(\frac{z_c \cdot s}{E}\right)^2 = \left(\frac{1.645 \times 8.94}{.5}\right)^2$ ≈ 865.10 rounds <u>up</u> to 866. *Keys:* [(] 1.645 [×] 8.94 [÷] .5 [)] [x^2] [ENTER]

Note: [PRGM] **SAMPSIZE** uses the <u>exact</u> value of z_c , 1.6449… getting 864.996 and rounding up to n = 865.

== **Begin 8.5** ==

11 A way to tell if two populations x_1 and x_2 are different <u>is</u> to examine the difference in μ_1 and μ_2 i.e. $(\mu_1 - \mu_2)$ <u>or</u> the difference in p_1 and p_2 i.e. $(p_1 - p_2)$.

If both sides of a cCI for $(\mu_1 - \mu_2)$ are negative, then we are c confident $(\mu_1 - \mu_2) < 0$ → $(\mu_1 < \mu_2)$.

If both sides of a cCI for $(\mu_1 - \mu_2)$ are positive, then we are c confident $(\mu_1 - \mu_2) > 0$ → $(\mu_1 > \mu_2)$.

If a cCI for $(\mu_1 - \mu_2)$ has opposite signs, then we cannot say which mean is larger.

The same applies to all of the above if we replace μ with p.

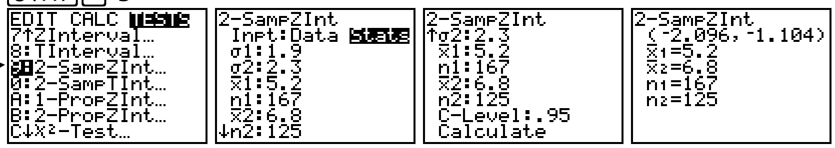
A c CI for $(\mu_1 - \mu_2)$ when σ_1 and σ_2 are <u>known</u> is $(\bar{x}_1 - \bar{x}_2) \pm z_c \cdot \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$. ← *A 2-Sample Z Interval.* *Page 447*

If n_1 and n_2 are both < 30, then the populations must be normal to use this formula.
If n_1 and n_2 are both > 29, then the populations do not need to be normal.

448, 9 Populations x_1 and x_2 have $\sigma_1 = 1.9$ and $\sigma_2 = 2.3$. Find a 95% CI for $(\mu_1 - \mu_2)$. $1 \equiv$ before. $2 \equiv$ after.

Samples are drawn with $n_1 = 167$, $\bar{x}_1 = 5.2$ and $n_2 = 125$ and $\bar{x}_2 = 6.8$.

STAT ◄ 9



→ A 95% CI for $(\mu_1 - \mu_2) = (-2.1, -1.1)$.

Both sides of the CI are negative → we are 95% confident that $(\mu_1 < \mu_2)$ → $(\mu_2 > \mu_1)$

The mean number of fish caught/day μ_2 after the fire is greater than the mean number of fish caught/day μ_1 before the fire.

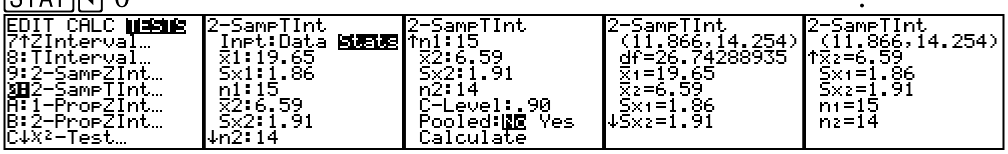
A c CI for $(\mu_1 - \mu_2)$ when σ_1 and σ_2 are unknown is found using (2-Samp TInt…). *Page 449.1.*

450, 10 Populations x_1 and x_2 are normal and have unknown σ_1 and σ_2. Find a 90% CI for $(\mu_1 - \mu_2)$.

Samples are drawn with $n_1 = 15$, $\bar{x}_1 = 19.65$, $s_1 = 1.86$ and $n_2 = 14$, $\bar{x}_2 = 6.59$, $s_2 = 1.91$

→ $\bar{x}_1 - \bar{x}_2 = 13.06$. 1→ wine

STAT ◄ 0



A 90% CI for $(\mu_1 - \mu_2) = (11.9, 14.3)$.

μ_1 is the mean brain activity of people that drank ½ liter of wine before sleeping.

μ_2 is the mean brain activity of people that drank no wine before sleeping.

Both sides of the CI are positive → we are 90% confident that $(\mu_1 > \mu_2)$.

(The mean brain activity of the drinking group) is greater than (the mean brain activity non-drinking group).

A c CI for $(p_1 - p_2)$ for two independent binomial experiments is $(\hat{p}_1 - \hat{p}_2) \pm z_c \cdot \sqrt{\dfrac{\hat{p}_1\hat{q}_1}{n_1} + \dfrac{\hat{p}_2\hat{q}_2}{n_2}}$. *Page 454*

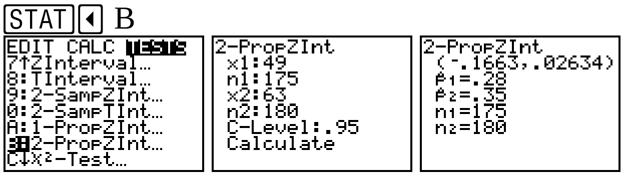
Where all of $n_1\hat{p}_1$, $n_1\hat{q}_1$, $n_2\hat{p}_2$, $n_2\hat{q}_2$, > 5 *A 2-Proposition Z-Interval.*

454, 11 Group 1 watched a comedy before sleep with $n_1 = 175$, and $r_1 = x_1 = 49$ had bad dreams.

Group 2 watched nothing before sleep with $n_2 = 180$, and $r_2 = x_2 = 63$ had bad dreams.

Find a 95% CI for $p_1 - p_2$.

STAT ◄ B



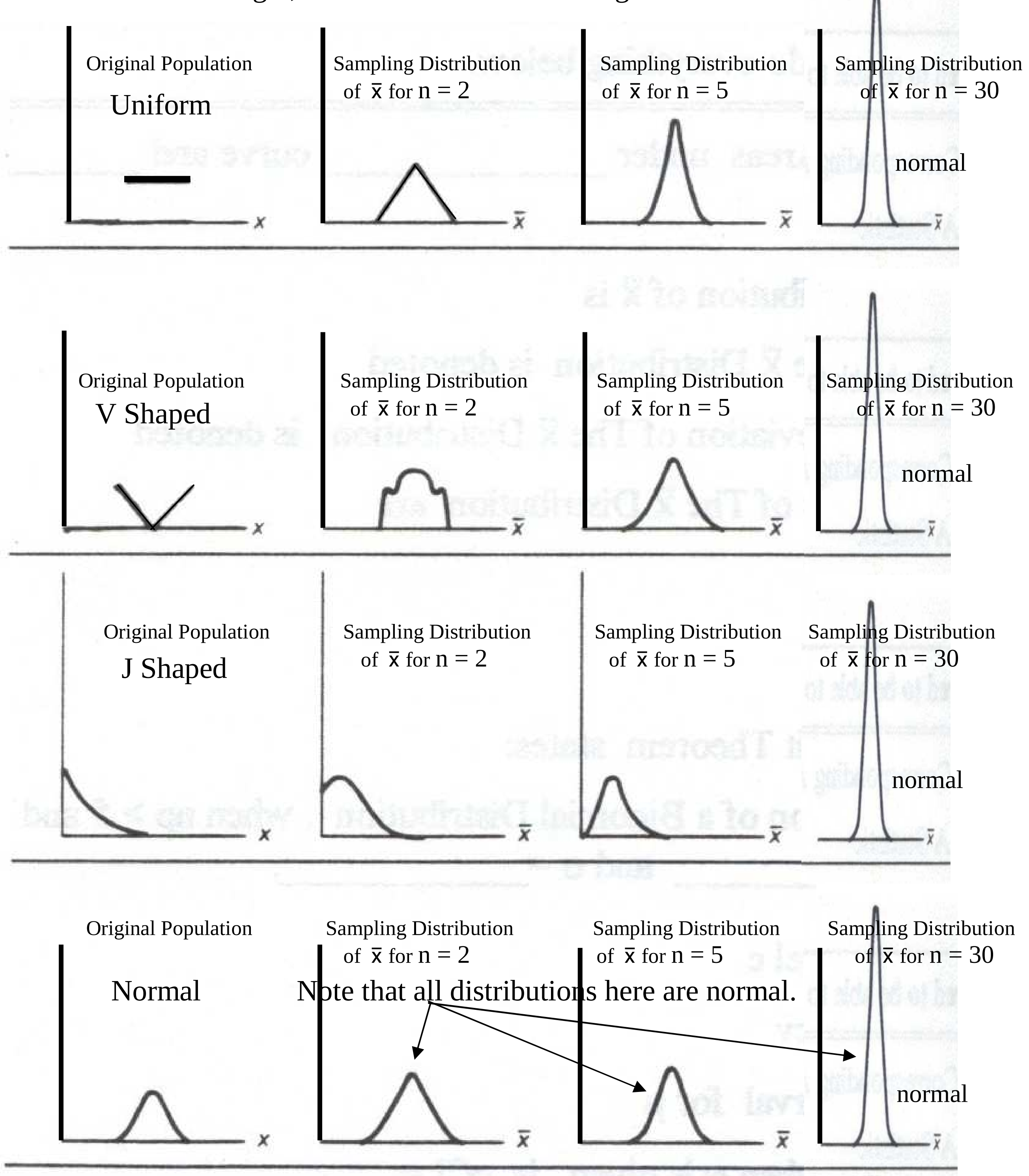
→ A 95% CI = (-17%, 3%).

Since the CI contains opposite signs, we cannot say that a comedy before sleep reduced bad dreams.

== End Block 3 ==

Sampling Distributions of $\bar{x}$ for 4 Different Populations and 3 Sample Sizes

Note that as n becomes larger, the $\bar{x}$ distributions tend to get closer to normal.



Here are the actual 18 test items as they will appear on Test L3.
Ten of these will be selected for the test but you are required to be able to do all of them.
The test item is in this font. *The answer is written in this font* or in a box

---

1 Complete: Corresponding Areas under *a normal curve* and *the standard normal* curve are equal.

2 Define: A Statistic *A Statistic is a number that is a descriptive measure of a sample.*

3 Complete: A sampling distribution of $\bar{x}$ is
a probability distribution formed by all the means $\bar{x}$ *of all the samples, all of size n, from a population.*

4 Complete: The Mean of The $\bar{x}$ Distribution is denoted $\boxed{\mu_{\bar{x}}}$.

5 Complete: The Standard Deviation of The $\bar{x}$ Distribution is denoted $\boxed{\sigma_{\bar{x}}}$.

6 Complete: Three Properties of The $\bar{x}$ Distribution are

a) *If the population is normal, then the* $\bar{x}$ *distribution is normal.*

b) $\boxed{\mu_{\bar{x}} = \mu}$. c) $\boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$.

7 Complete: The Central Limit Theorem states:

a) *The* $\bar{x}$ *distribution from any population is usually normal when n > 29,*

b) $\boxed{\mu_{\bar{x}} = \mu}$. c) $\boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$.

1 Define: The Confidence Level c *The Confidence Level c is the reliability of the confidence interval.*

2 Define: The Critical Value cv
The Critical Value cv, denoted z_c *or* t_c, *is the right boundary of the central area c.*

3 Define: A c Confidence Interval for μ
A c Confidence Interval for μ *is an interval cCI that is estimated to contain* μ *with a c level of confidence.*

4 Complete: The cCI for μ when σ is given is cCI = $\boxed{\left(\bar{x} \pm z_c \cdot \frac{\sigma}{\sqrt{n}}\right)}$

5 Complete: The cCI for μ when σ is **not** given is cCI = $\boxed{\left(\bar{x} \pm t_c \cdot \frac{s}{\sqrt{n}}\right)}$

---

6 Complete: The Maximum Error E of Estimate of the cCI is E = $\boxed{cv \cdot \frac{SD}{\sqrt{n}}}$

7 Complete: 1) Student's t Distribution is used when $\boxed{\sigma \text{ is } \underline{\text{unknown}}}$.

2) If n < 30, then *the population must be "normal" to use the t distribution.*

8 Complete: The Number of Degrees of Freedom for a t distribution is d.f. = $\boxed{n - 1}$.

9 Complete: The Point Estimate $\hat{p}$ for p in a binomial experiment with n trials and r successes is $\boxed{\hat{p} = \frac{r}{n} = \frac{x}{n}}$

10 Complete: The sample size n for a cCI with maximum error E is n = $\boxed{\left(\frac{z_c \cdot s}{E}\right)^2 \text{ rounded up}}$

11 To tell if two populations x_1 and x_2 are different, examine the difference in μ_1 and μ_2 i.e. $(\mu_1 - \mu_2)$ <u>or</u> the difference in p_1 and p_2 i.e. $(p_1 - p_2)$

If both sides of a cCI for $(\mu_1 - \mu_2)$ are negative, then we are c confident that $\boxed{\mu_1 < \mu_2}$.

If both sides of a cCI for $(\mu_1 - \mu_2)$ are positive, then we are c confident that $\boxed{\mu_1 > \mu_2}$.

If a cCI for $(\mu_1 - \mu_2)$ has opposite signs, then *we cannot say which mean is larger.*

You are required to be able to do everything below.

1 Complete: Corresponding Areas under _______________ curve and ____________________ curve are equal.

2 Define: A Statistic

3 Complete: A sampling distribution of $\bar{x}$ is

4 Complete: The Mean of The $\bar{x}$ Distribution is denoted

5 Complete: The Standard Deviation of The $\bar{x}$ Distribution is denoted

6 Complete: Three Properties of The $\bar{x}$ Distribution are
a)
b)
c)

7 Complete: The Central Limit Theorem states:

1 Define: The Confidence Level c

2 Define: The Critical Value cv

3 Define: A c Confidence Interval for μ

4 Complete: The cCI for μ when σ is given is cCI =

5 Complete: The cCI for μ when σ is **not** given is cCI =

6 Complete: The Maximum Error E of Estimate of the cCI is E =

7 Complete: 1) Student's t Distribution is used when
2) If $n < 30$, then

8 Complete: The Number of Degrees of Freedom for a t distribution is d.f. =

9 Complete: The Point Estimate $\hat{p}$ for p in a binomial experiment with n trials and r successes is

10 Complete: The sample size n for a cCI with maximum error E is n =

11 To tell if two populations x_1 and x_2 are different, examine the difference in μ_1 and μ_2 i.e. $(\mu_1 - \mu_2)$
<u>or</u> the difference in p_1 and p_2 i.e. $(p_1 - p_2)$

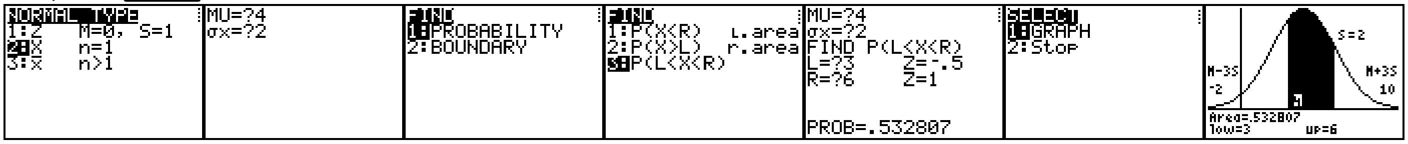
If both sides of a cCI for $(\mu_1 - \mu_2)$ are negative, then we are c confident that …

If both sides of a cCI for $(\mu_1 - \mu_2)$ are positive, then we are c confident that …

If a cCI for $(\mu_1 - \mu_2)$ has opposite signs, then …

6.3

330, 1 PRGM **NORMAL**

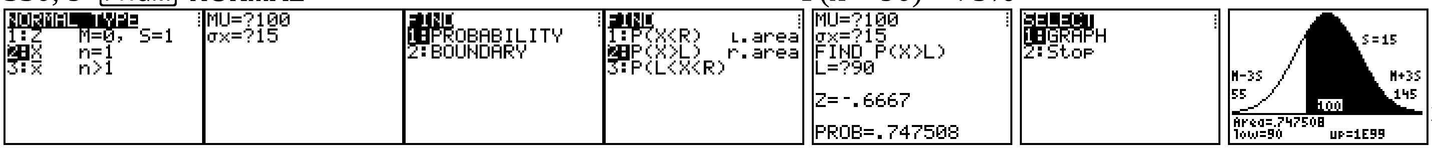


x

Note: In a continuous distribution P(3 < x < 6) = P(3 ≤ x ≤ 6). P(3 < x < 6) ≈ 53%

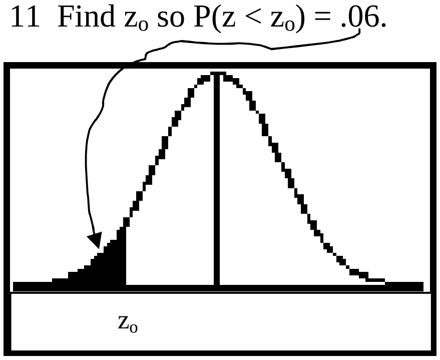
330, 9 PRGM **NORMAL**

P(x > 90) ≈ 75%

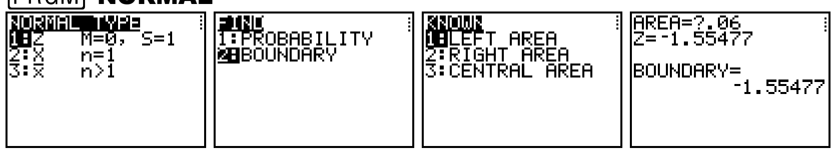


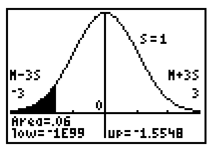
x

330, 11 Find z_o so $P(z < z_o) = .06$.



PRGM **NORMAL**

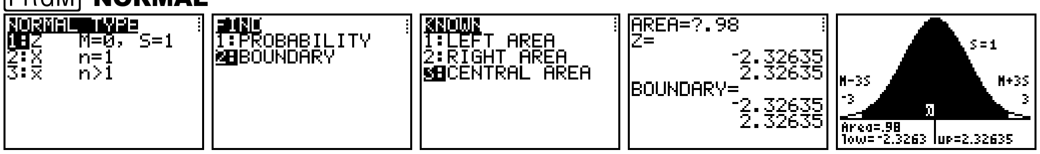




z

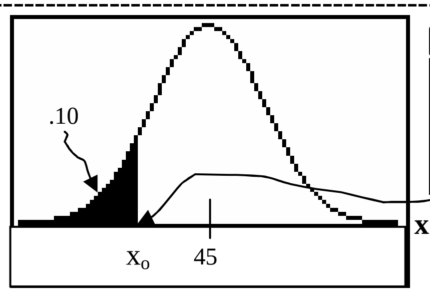
$z_o \approx -1.555$ $P(z < -1.555) \approx .06$

331, 19 PRGM **NORMAL**

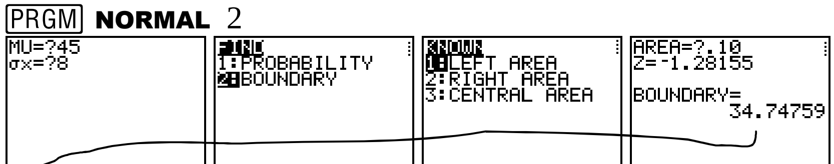


z

332, 29b



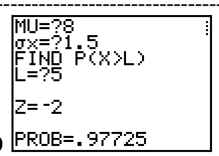
PRGM **NORMAL** 2



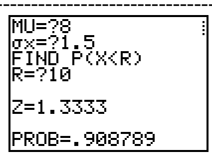
If the batteries are guaranteed for 35 months, the company will only have to replace about 10.6% of them.

333, 31 PRGM **NORMAL** 2

b

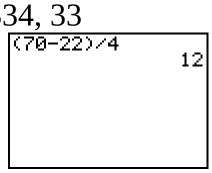


c

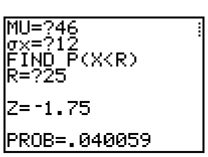


334, 33

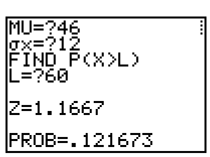
a



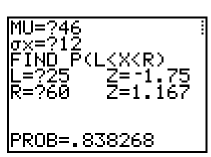
b



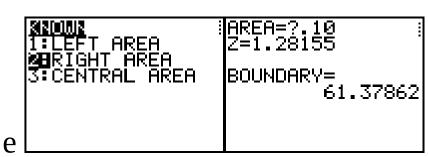
c



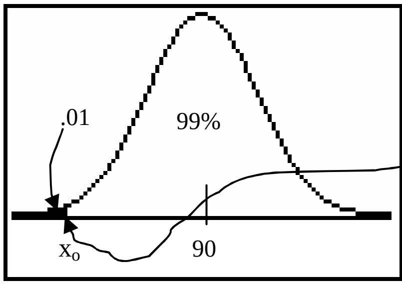
d



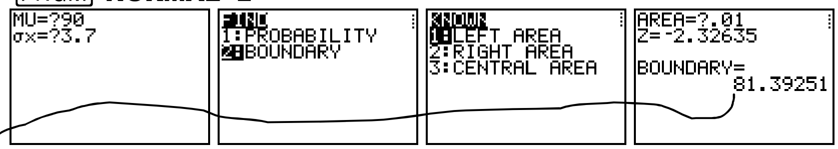
e



334, 35a

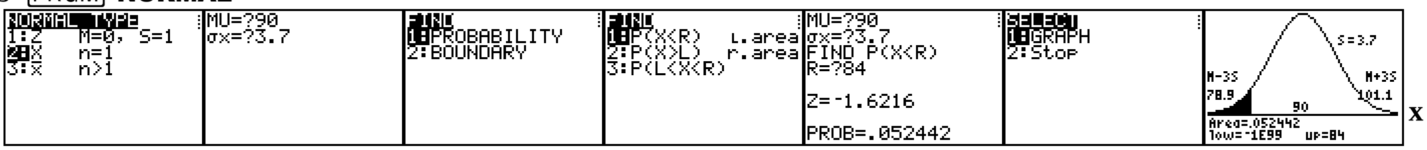


PRGM **NORMAL** 2



Guarantee a chip for 81 months and there is a 99% probability that it will last beyond the guarantee period.

b PRGM **NORMAL**

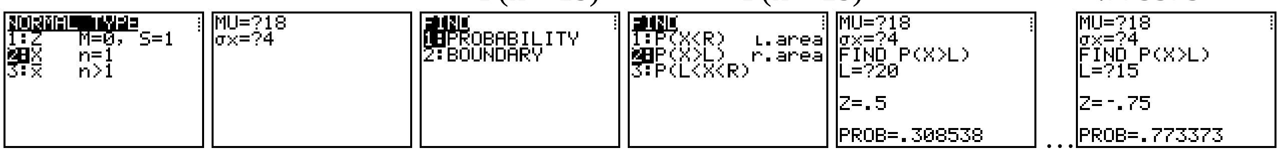


Guarantee the chip for 84 months and there is a 5% probability that it will malfunction in the guarantee period.

334, 35 (c) \$50,000,000 × .0524422 = \$2,622,110 is the expected loss to the company.

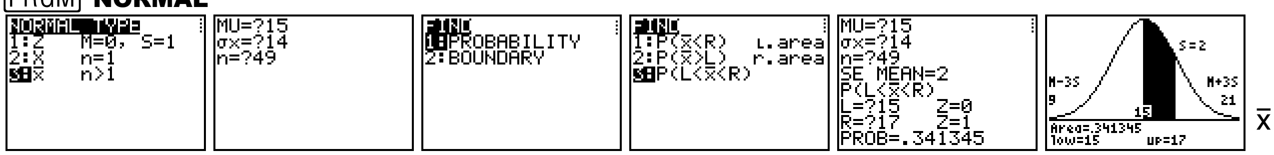
334, 35 (d) Profit = 3,000,000 – 2,622,110 = \$377,890. *(The books answer is the result of rounding.)*

335,39a $P(x > 20 \mid x > 15) = \dfrac{P((x > 20 \text{ and } (x > 15))}{P(x > 15)} = \dfrac{P(x > 20)}{P(x > 15)}$ → *see below* → $\dfrac{.308538}{.773373} \approx .4 = 40\%$



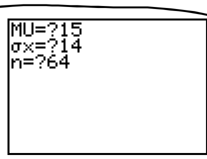
7.2

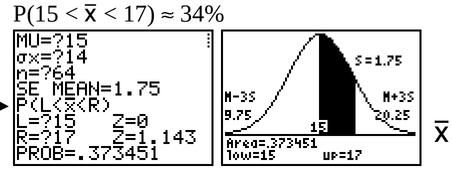
373, 1a PRGM **NORMAL**



b

P(15 < x̄ < 17) ≈ 34%





c In the (a) distribution, 15 and 17 have z-scores of 0 and 1.

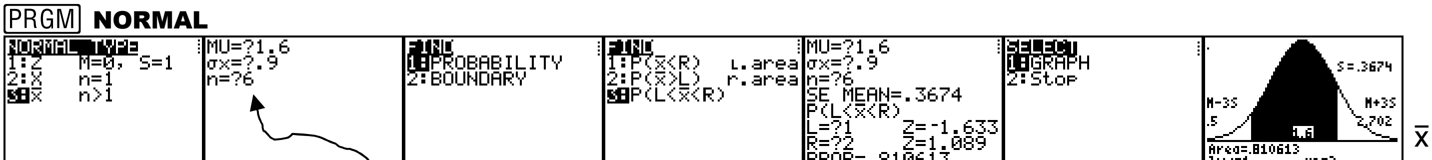
In the (b) distribution, 15 and 17 have z-scores of 0 and 1.14. There is more area between 0 and 1.14.

$\sigma_{\bar{x}}$ = 1.75 in b and is smaller than $\sigma_{\bar{x}}$ = 2 in a. b has a narrower distribution, so the area between 15 and 17 is bigger.

375, 11 x is the **monthly percent return** of the mutual fund. The fund has stocks from 250 companies. This is a large sample of all of the companies in the world. Because x is a large sample of averages, it is an x̄ distribution that is normal. For x, μ = 1.6 and σ = .9. *(Both are percents.)*

(b) n = 6. This sample is small, n = 6, but it is from a normal distribution so its distribution x̄ is normal.

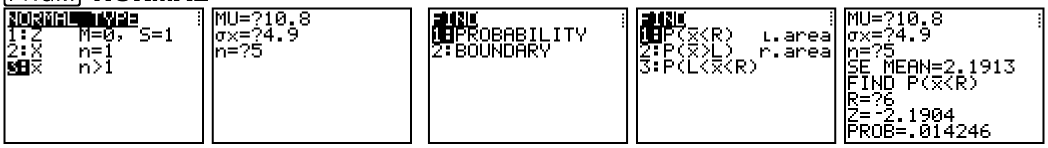
PRGM **NORMAL**



(c) is done like (b) except n = 24.

P(1 < x̄ < 2) = 81%

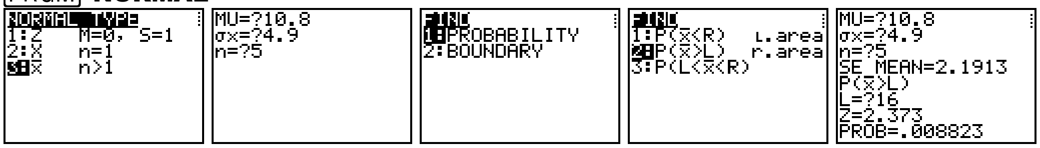
376, 13 b PRGM **NORMAL**



$P(\bar{x} < 6\%) = .014$

With a probability this small, it is unlikely that the mean was really 10.8%. The market is weaker than 10.8%.

PRGM **NORMAL**

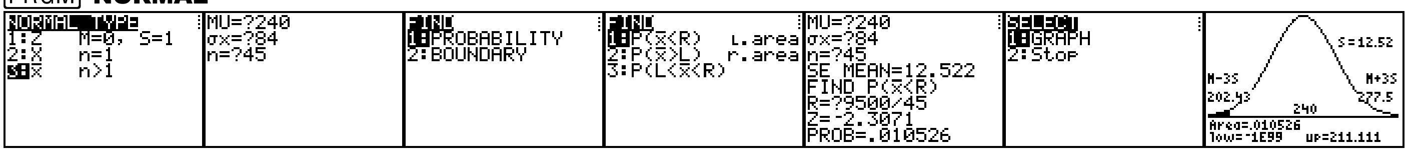


$P(\bar{x} > 16\%) = .009$

It is very unlikely that the mean was really 10.8%. The market is heating up and the yield is more than 10.8%.

378, 19 (a) If the total weight w of 45 m^3 is 9500g, then the *average* weight $\bar{x}$ of 1 m^3 is $\frac{9500}{45} \approx 211.$

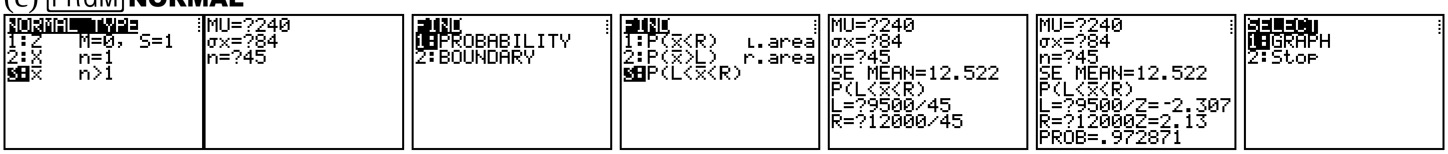
PRGM **NORMAL**

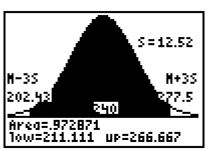


$\bar{x}$

$$P(w < 9500) = P\left(\bar{x} < \frac{9500}{45}\right) \approx .011$$

(c) PRGM **NORMAL**





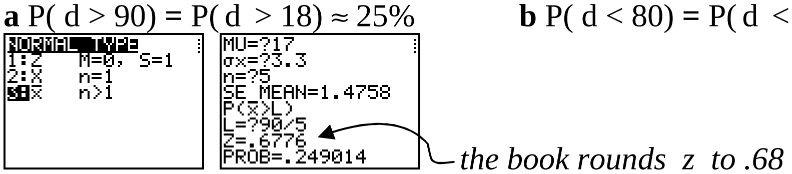
$\bar{x}$

$$P(9500 < w < 12000) = P\left(\frac{9500}{45} < \bar{x} < \frac{12000}{45}\right) \approx 97\%$$

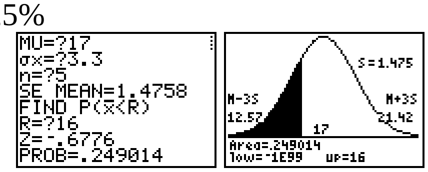
378, 21 (a) If the total distance d in 5 years is 90 ft, then the *average* distance $\bar{d}$ in 1 year is $\frac{90}{5} = 18$ ft.

(b) If the total distance d in 5 years is 80 ft, then the *average* distance $\bar{d}$ in 1 year is $\frac{80}{5} = 16$ ft.

a $P(d > 90) = P(\bar{d} > 18) \approx 25\%$

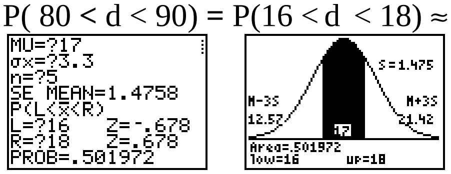


b $P(d < 80) = P(\bar{d} < 16) \approx 25\%$



$\bar{d}$

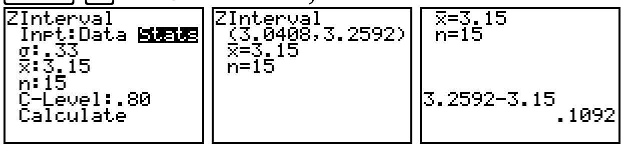
c $P(80 < d < 90) = P(16 < \bar{d} < 18) \approx 50\%$



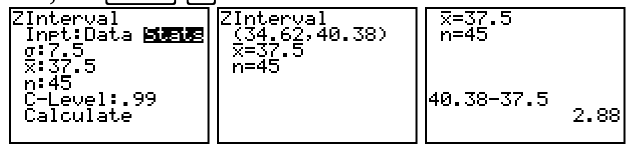
$\bar{d}$

8.1 --

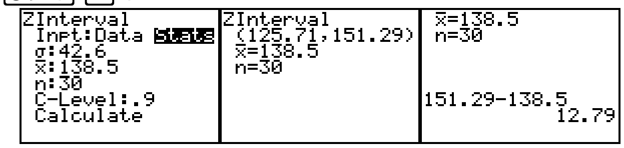
408, 1 STAT ◄ 7 σ is known, so a ZInterval is correct.

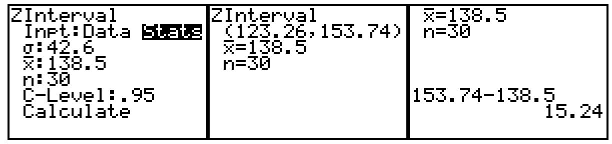


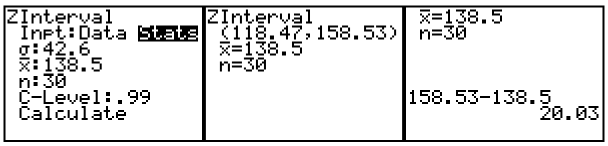
408, 3 STAT ◄ 7



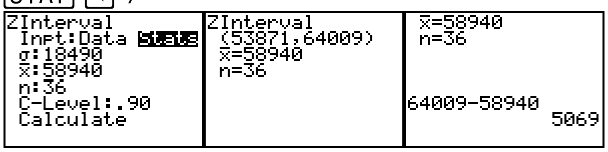
409, 5 STAT ◄ 7

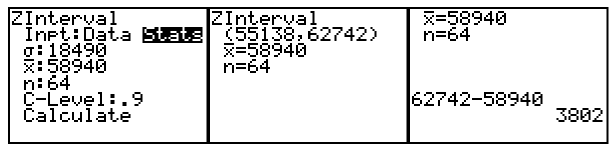


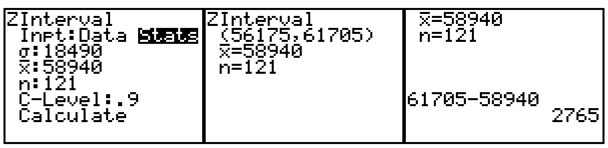




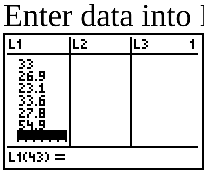
410, 7 STAT ◄ 7

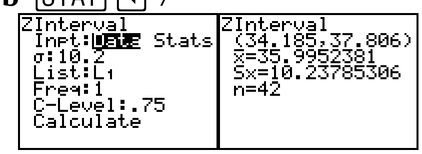


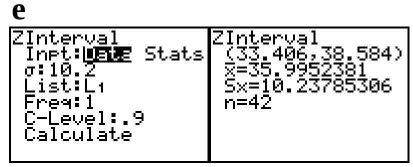




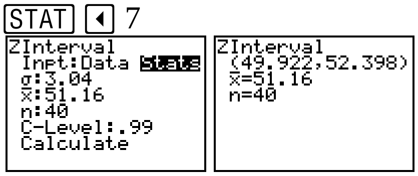
410, 9 **a** Enter data into L1 **b** STAT ◄ 7 **e**







411,11**c** STAT ◄ 7



99%CI = (49.92, 52.40)

8.2 --

420, 1 PRGM **cv**

420, 3 PRGM **cv**

420, 5 Enter the 9 data values into L1. STAT ◄ 8 σ is unknown, so a t interval is correct.

```
L1     L2     L3    1
1272
1268
1316
1275
1317
1275
L1(10) =
```

```
TInterval
 Inpt:Data Stats
 List:L1
 Freq:1
 C-Level:.90
 Calculate
```

```
TInterval
 (1249.3,1295.1)
 x̄=1272.222222
 Sx=36.9282337
 n=9
```

420, 7

```
L1     L2     L3    1
68     ------ ------
104
128
122
80
64
L1(7)=
```

```
TInterval
 Inpt:Data Stats
 List:L1
 Freq:1
 C-Level:.75
 Calculate
```

```
TInterval
 (74.685,107.31)
 x̄=91
 Sx=30.71807286
 n=6
```

421, 9

```
TInterval
 Inpt:Data Stats
 x̄:79.25
 Sx:5.33
 n:6
 C-Level:.8
 Calculate
```

```
TInterval
 (76.039,82.461)
 x̄=79.25
 Sx=5.33
 n=6
```

421, 11 b c

```
TInterval
 Inpt:Data Stats
 x̄:9.95
 Sx:1.02
 n:10
 C-Level:.999
 Calculate
```

```
TInterval
 (8.4079,11.492)
 x̄=9.95
 Sx=1.02
 n=10
```

8.3 ---

433, 1 STAT ◄ A

```
1-PropZInt
 x:39
 n:62
 C-Level:.95
 Calculate
```

```
1-PropZInt
 (.50879,.74927)
 p̂=.6290322581
 n=62
```

433, 3 STAT ◄ A

```
1-PropZInt
 x:1619
 n:5222
 C-Level:.99
 Calculate
```

```
1-PropZInt
 (.29355,.32652)
 p̂=.3100344696
 n=5222
```

434, 5 STAT ◄ A

```
1-PropZInt
 x:3139
 n:5792
 C-Level:.99
 Calculate
```

```
1-PropZInt
 (.52509,.55882)
 p̂=.5419544199
 n=5792
```

435, 15 STAT ◄ A

```
1-PropZInt
 x:628
 n:730
 C-Level:.95
 Calculate
```

```
1-PropZInt
 (.83512,.88542)
 p̂=.8602739726
 n=730
```

```
 p̂=.8602739726
 n=730

.88542-.86027
           .02515
```

436, 17 STAT ◄ A

```
1-PropZInt
 x:250
 n:1000
 C-Level:.95
 Calculate
```

```
1-PropZInt
 (.22316,.27684)
 p̂=.25
 n=1000
```

```
 p̂=.25
 n=1000

.27684-.25
         .02684
```

8.4 ---

442,1 PRGM **SAMPSIZE**

```
c=?.95
S=?44
E=?10
n=
           75
         Done
```

442, 5 $n_1 = 56$ ➔ s = 26.58 PRGM **SAMPSIZE**

```
c=?.90
S=?26.58
E=?4
n=
          120
         Done
```

120 – 56 = 64

We need 64 <u>more</u> weights to be 90% confident that the average weight of the 120 players will be within 4 pounds of the actual average of all of the players.

444, 15 $n_1 = 167$ ➔ s = 3.8. E = .5 (30 seconds = .5 minutes) PRGM **SAMPSIZE**

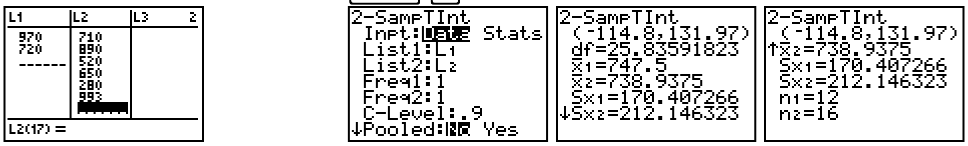
```
c=?.99
S=?3.8
E=?.5
n=
          384
         Done
```

384 – 167 =217.

Get 217 <u>more</u> customers to be 99% confident that the mean of the 384 times will be no more than 30 seconds from the true mean.

8.5 --

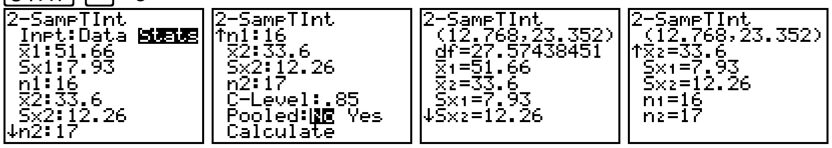
457, 1 Enter data into L1 and L2 [STAT] [◄] 0



←The TI gives better results.

The CI for ($\mu_1 - \mu_2$) has opposite signs, so we cannot say with 90% confidence which mean is larger.

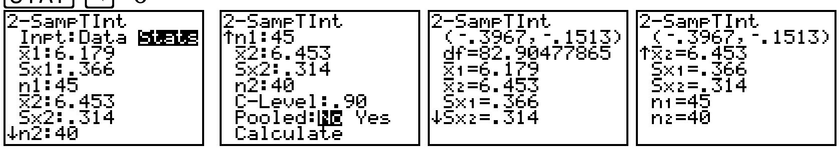
458, 3 b [STAT] [◄] 0



← The TI gives better results.

Both sides of the CI for ($\mu_1 - \mu_2$) are positive. We are 85% confident that ($\mu_1 > \mu_2$).

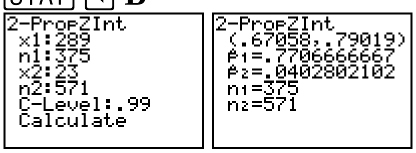
459, 5 b [STAT] [◄] 0



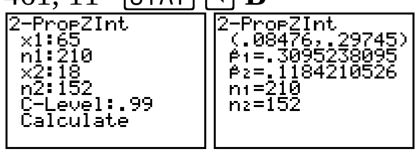
← The TI gives better results.

Both sides of the CI for ($\mu_1 - \mu_2$) are negative. We are 90% confident that ($\mu_1 < \mu_2$).

460, 7 [STAT] [◄] **B**



461, 11 [STAT] [◄] **B**



Both sides of the CI for ($p_1 - p_2$) are positive. We are 99% confident that ($p_1 > p_2$).
