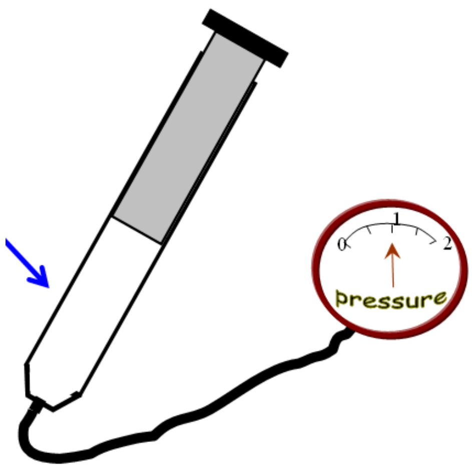
The Ideal Gas Law

This activity builds on the *The Behavior of Gases* laboratory activity that you recently completed. There you explored the behavior of gases in mainly a qualitative way, just beginning analysis of quantitative results when you explored the pressure-volume relationship. Now we are going to examine the gas variables (pressure, volume, temperature, and amount as moles) quantitatively in an interactive Excel spreadsheet at http://academic.pgcc.edu/psc/chm101/ideal_gas.

1. On the set-up below the arrow points to a volume of 10 mL of air at 1.0 atm. How will pressure vary when the volume is changed? Now before starting, what is the temperature of the trapped air? Is the amount of air in the syringe constant? Assuming that our syringe does not leak, we have a closed system or a constant amount of air trapped at laboratory temperature (25 °C or 298K).



Now, how will pressure vary when the volume is changed? Sketch the relationship and label the axes in the space to the right.

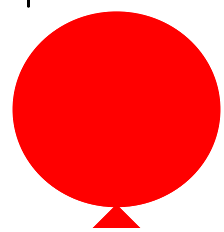
Open the Ideal Gas Law spreadsheet and click on the P-V tab. This plot should look familiar. Remember that PV = k. What happens if temperature increases? Try it out and sketch and label on the plot above.

What is happening on a microscopic level as the pressure on a gas increases? View animation #1 at http://academic.pgcc.edu/psc/chm101/ideal_gas.

What happens if the number of moles of gas in the syringe decreases? Why?

How do the PV values compare when the error is zero? Add a little error to the data using the error slider. What do you notice about the graph? What happens to k (PV)?

2. When you warmed the balloon over the hot water on the hot plate what happened to the balloon?



What caused this change? (Consider what is happening on a microscopic level.) View animation #2 at http://academic.pgcc.edu/psc/chm101/ideal_gas.

What variables were constant for the balloon as you heated it?

How does volume vary as the temperature is raised? Click on the V-T (C) tab to see the graph of volume as a function of Celsius temperature. Sketch and label the graph.

How does the V/T ratio vary for this plot when the error is zero?

How does changing the pressure influence the graph? Sketch and label the effect.

What is the numerical value of the x-intercept? What does this mean?

This is the basis for the Kelvin temperature scale. When the projected or extrapolated gas volume reaches zero, the temperature at which this occurs is defined as <u>absolute zero</u>. This is how the Kelvin temperature scale was developed.

How does error influence the VT result?

How could you transform this relationship (have a y-intercept of zero) to determine a direct proportion between volume and temperature with no other factors?

Sketch a graph of volume as a function of temperature, where temperature is converted to Kelvin. The transformation of the temperature data is the conversion from Celsius to Kelvin temperature: K = °C + 273. Click on the V-T (K) tab to view this plot.

How does the V/T ratio vary for this plot when the error is zero?

Why does the data have a lower temperature limit?

3. Imagine you blow air into a balloon. What happens to the balloon as the amount of air increases?

Is there a direct relationship between volume and amount of air?

Click on the V-n tab. Sketch and label the axes for volume as a function of moles.

How does changing the pressure influence the graph? Sketch and label the effect.

How does changing the temperature influence the graph? Sketch and label the effect.

How does the V/n ratio vary for this plot? Why?

From the variation of pressure versus volume, volume versus temperature (in Kelvin), and volume versus moles, we can combine these and get the <u>ideal gas law</u>:

$$PV = k$$

$$\frac{V}{T} = k' \longrightarrow \frac{PV}{nT} = k''' \longrightarrow PV = nRT$$

$$\frac{V}{n} = k'' \qquad \text{let } k''' = R$$

where R is referred to as the <u>gas constant</u>.

4. We define a reference or standard temperature (0°C or 273.15 K) and pressure (1.00 atmosphere) and refer to these as STP. From experimentation it is found that 1.00 mole of <u>any</u> gas (H_2, C_4H_{10}, or Rn) occupies a volume of 22.4 L at STP. Using these experimental conditions, find the numerical value of the gas constant, R, and its units.

5. Using the animations and the ideal gas law, explain how pressure and temperature are related. Sketch and label a graph of pressure as a function of temperature (in kelvin) on the axes.

What variables must be constant for this relationship to hold?

How does changing the volume influence the graph? Sketch and label on the graph.

How does changing the number of moles influence the graph? Sketch and label on the graph. Explain on a microscopic level why this relationship exists.

How does the P/T ratio vary for this plot?

6. When you warmed the balloon on the hot plate, the volume increased. What about the amount of gas in the balloon, did it change?

If the balloon is a closed system (has no leaks), how did the density change for the balloon as temperature rose?

What is the mathematical relationship between gas density and temperature?

Explain what is happening on a microscopic level to explain this relationship.

7. Why does a helium-filled balloon rise?

Let's consider how gas density varies as a function of the molar mass of the gas. For the same amount of gas at constant temperature, pressure and volume, how is density going to vary with the molar mass of the gas?

Click on the D-MM tab to see the relationship for the Group 8A or noble gases. Sketch and label the graph.

How does changing the pressure influence the graph? Sketch and label on the plot. Why does this happen?

How does changing the temperature influence the graph? Sketch and label on the plot.

How does the D/MM ratio vary?

Now derive an equation that expresses density as a function of molar mass and includes the influence of temperature and pressure.

The equation you just derived from the experimental data can be obtained by substituting and manipulating the ideal gas law:

$$PV = nRT$$

$$n = \frac{m}{MM} \quad \rightarrow \quad PV = \frac{m}{MM}RT \quad \rightarrow \quad D = \frac{m}{V} = \frac{P(MM)}{RT}$$

$$D = \frac{m}{V}$$

8. Suppose we wanted to calculate the density of air at 1.00 atm and 298K, we would need the molar mass of air. Now air is a mixture, not a compound, so how do we get a molar mass for air? Air, at 1.00 atm of pressure or at sea level, is mixture with a fixed composition – 80% N_2 and 20% O_2. Click on the air tab to calculate the molar mass of air. Your result has to be between 28 g/mole for pure N_2 and 32 g/mole for pure O_2.

MM_{air} = ________

In the future if greenhouse warming is increased by addition of CO_2 to the atmosphere, how will the MM of air change?

If water vapor increases in a warm environment, how will the MM of air change?

Calculate the density of air at 1.00 atm and 298K.

Hot air balloons rise. Now, based on what you have learned, explain why.

Here is another useful relationship derived from the ideal gas law for a trapped amount of gas (n = constant) or a closed system. This is called the <u>combined gas law</u>.

$$\frac{PV}{T} = nR = k'''' \quad \rightarrow \quad \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

9. Consider a 1.2-L balloon that travels from Largo, MD at 27°C and 1.00 atm to Denver, CO (the mile high city) where it is 15°C and 0.80 atm. Calculate the new volume of the balloon.

10. Now what about the velocity of a container of argon atoms at a constant temperature, would they all have the same velocity or different velocities?

Go to animation #3 at http://academic.pgcc.edu/psc/chm101/ideal_gas, which is a Java applet. Press the "setup" button to get 500 particles all at the same temperature, which actually is the same energy. Notice they are all green. Now press the "go" button. One particle has a gray tracer tail, so that you can follow its path. On starting the applet, what happens to the energy or velocity of the particles?

Are they all moving with the same velocity? Why or why not?

What does the tracer tail of the one particle show?

The random, rapid, chaotic motion of the particles leads to collisions. Energy is exchanged in a collision and some particles lose energy and move slower, while others gain energy and move faster. Now view the velocity tab on the Excel spreadsheet.

Let's consider 1.00 mole of argon gas at 25°C or 298K. How would you describe the velocity of the argon atoms?

How does 1.00 mole of He at 298K compare to argon at 298K?

How about 1.00 mole of Ar at 500K compared to Ar at 298K?

How does increasing the temperature influence the distribution of velocities?

How does increasing the molar mass influence the distribution of velocities?

In simple terms, the temperature (in Kelvin) of the gas influences the average kinetic energy of the particles:

$$KE_{avarage} = \frac{3}{2}RT = \frac{1}{2}(MM)v^2_{average}$$

where $v_{average}$ is the average velocity of the particles. The distribution of the velocities of the particles then depends on the temperature and the molar mass.

11. View animation #4 at http://academic.pgcc.edu/psc/chm101/ideal_gas. The two gases are both at the same temperature.

How does the kinetic energy compare?

How does the velocity compare? Why?

How could you get both gases to have the same velocity?

<u>More Practice Problems</u>

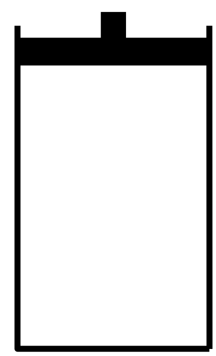
12. An empty aerosol can has a pressure of 1.0 atm at 25°C. What temperature would bring the can to its highest rated pressure of 4.5 atm?

13. A breath of air has a volume of 500 mL at 37°C and 1.00 atm. How many moles of air in a breath? How many total molecules (N_2 + O_2) are contained in this breath?

14. Calculate the density of radon, Rn, at 25°C and 1.00 atm. Air has a density of 1.2 g/L at these conditions. Would radon accumulate in your basement or attic? Justify your answer.

<u>Stop at this point in the activity</u>. The remainder of this activity requires some knowledge of liquids and we will come back to it later.

15. The pressure on a gas was increased by adding mass to the top of the container and compressing the volume, while temperature was maintained constant. Plot the following data on a graph. What is the dependent variable?



P (atm)	V (mL)
1.0	1000
1.5	667
2.0	500
2.5	400
3.0	333
3.5	286
4.0	250
4.5	222
4.5	0.22
5.0	0.22
5.5	0.22
6.0	0.22

Explain what has happened to the gas as pressure was increased.

Why can this change in volume happen?

16. In ideal gases there are no intermolecular forces operating. Why?

For a liquid to form, the molecules must get close enough for intermolecular forces (IMF) to operate. In gases the particles are too far apart and have too much energy (moving at too fast a velocity) to interact with one another. So to liquefy a gas, the pressure is increased and usually the temperature is lowered as well.

Why does lowering the temperature help liquefy a gas?

<u>What makes a gas behave as an ideal gas?</u>

These two assumptions must be met for a gas to be ideal:

- There are no molecular interactions between the particles (atoms or molecules) and;
- the gas particles do not take up any appreciable volume or space in the container.

If these conditions apply, the gas behavior can be accurately predicted using the gas laws we have developed already. However, we find that some gases deviate from expected behavior. Now let's see how well these assumptions are met for gases. To evaluate this we can draw a graph of PV/RT vs. P for 1.00 mole of gas.

17. How does PV/RT behave for an ideal gas? PV/RT = ______

What would a graph of PV/RT vs. P look like?

Now go to the PV over RT ratio tab to see. If a gas is ideal, the volume should change as the pressure changes and the plot should be a straight line with slope = 0 and a y-intercept of one, since PV/RT = 1 for n = 1.00 mole.

18. From the graph, can you conclude that all gases behave ideally at all pressures? Why or why not?

If they are not ideal, what do you think is happening on a microscopic level with regard to the assumptions as pressure increases?

Now we have a problem. How do we predict the behavior of gases if they behave in a non-ideal or "real" manner?

The van der Waals equation for real gases is a modification of the ideal gas law that corrects for molecular interactions and the volume occupied by the molecules.

The equation is given below:

$$\left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT$$

where "a" corrects for molecular attractions and "b" corrects for the actual volume of the molecules or excluded volume.

19. What happens to van der Waals equation if a = 0 and b = 0?

The table given below lists the values of "a" and "b" for a number of gases at room conditions.

Gas	"a" (L^2-atm/$mole^2$)	"b" (L/mole)
He	0.034	0.024
Ne	0.21	0.017
H_2	0.24	0.027
O_2	1.36	0.032
N_2	1.39	0.039
Cl_2	6.49	0.056
CO	1.49	0.040
CO_2	3.59	0.043
NH_3	4.17	0.037
CH_4	2.25	0.043
C_2H_2	4.39	0.051

From: Adamson (1973) *A Textbook of Physical Chemistry*

20. Click on the real gas tab and explore the behavior of some of the gases listed in the table above. How do most gases behave around 298 K and 1.0 atm?

Under what two conditions do the ideal gas assumptions <u>not</u> hold? Why?